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Discrete Signal Reconstruction by Sum of Absolute Values

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Discrete Signal Reconstruction

Alphabet set: $X = \{r_1, r_2, \dots, r_L\}$
Probability: $P(r_k) = p_k, \quad k = 1, 2, \dots, L$

Reconstruction problem: Given linear measurements

$$y = \Phi x \in \mathbb{C}^m$$

of the original signal $x \in X^n$ where $n > m$, find the original x.

The idea

If $x \in X^n$, then $x - r_k$ is sparse.

Sum-of-Absolute-Values Optimization (SOAV)

$$\min_{z \in \mathbb{R}^n} \sum_{k=1}^{L} p_k \|z - r_k\|_1 \ \text{s.t.}\ y = \Phi z$$

$$\min_{z \in \mathbb{R}^n} \sum_{k=1}^{L} p_k \|z - r_k\|_1 \ \text{s.t.}\ \|y - \Phi z\|_2 \le \epsilon$$

$$\min_{z \in \mathbb{R}^n} \sum_{k=1}^{L} p_k \|z - r_k\|_1 + \lambda \|y - \Phi z\|_2^2$$

Dynamical Systems

Differential equation

$$\frac{dx(t)}{dt} = Ax(t) + Bz(t), \quad t \ge 0 \ (*)$$

Boundary Conditions: $x(0) = x_0, x(T) = 0$

Constraint: $|z(t)| \le 1, \ \forall t \in [0, T]$

Discreteness: $z(t) \in \{r_1, r_2, \dots, r_L\}$ a.e. $t > 0$

Problem
Find a function $z(t)$ on $[0, T]$ that satisfies the boundary conditions, constraints, and discreteness under the dynamics of (*).

(Theorem) Minimizing

$$J = \sum_{k=1}^{L} \int_0^T |z(t) - r_k| \, dt$$

gives a solution (under mild assumptions).

After time discretization, the (infinite-dimensional) optimization is reduced to SOAV.

Binary image reconstruction

Example

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} z(t)$$